Chapter 2 Homework Problems:

1. MWIR radiation covers the 3-5μ portion of the EM spectrum. What energy range does this correspond to, in eV.

$$Energy = \text{hf} = \frac{\text{hc}}{l} = \frac{4.136\times10^{-15}\,eV\,s \bullet 3\times10^{8}\,\frac{m}{s}}{l} = \frac{1.238\times10^{-6}\ eV\ m}{l\,(m)}$$

Energy = 0.413 eV, 0.248 eV at 3 and 5 micron, respectively

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2. What frequency is an x-band radar? What wavelength?
   3 cm, 1×10^{10} Hz

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3. What is the ground state energy for an He+ ion, in eV. (Note, Z=2)

$$Energy = -\frac{13.6\ \text{eV} \bullet \text{Z}^2}{\text{n}^2};\ \text{Z} = 2,\ \text{n} = 1$$

Energy = -54.4 eV (note, the minus sign is important, it means the electron is trapped.)

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4. Calculate the energy (in eV), frequency (in Hz), and wavelength (in meters, microns, and nano-meters) for the n=4 to n=2 transition in a hydrogen atom (note, Z=1). This is the Balmer-β transition.

$$Energy = -\frac{13.6\ \text{eV} \bullet \text{Z}^2}{\text{n}^2};\ \text{Z} = 1,\ \text{n} = 2,\ 4$$

The energy levels involved here are n = 2 (E = -3.39 eV), and n = 4 (E = -0.85).
For an emitted photon, we get

$$\Delta E = \text{E}_4 - \text{E}_2 = -0.85 - (-3.39) = 2.54\ \text{eV}$$

$$Energy = \text{hf};\ \text{f} = \frac{\text{Energy}}{\text{h}} = \frac{2.54\ eV}{4.136\times10^{-15}\,eV\ s} = 6.156\times10^{14}\ \text{Hz}$$

$$l = \frac{\text{c}}{\text{f}} = \frac{3\times10^{8}\ \frac{m}{s}}{6.156\times10^{14}\ \text{Hz}} = 4.87\times10^{-7}\ \text{meters}$$

or 4861 Angstroms officially, the Balmer-β line.

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5. Student exercise: check long wavelength behavior - use the fact that for small x: $e^x - 1 \approx x$ to get rid of the exponential term in the denominator. Also, which term dominates L for small wavelength?

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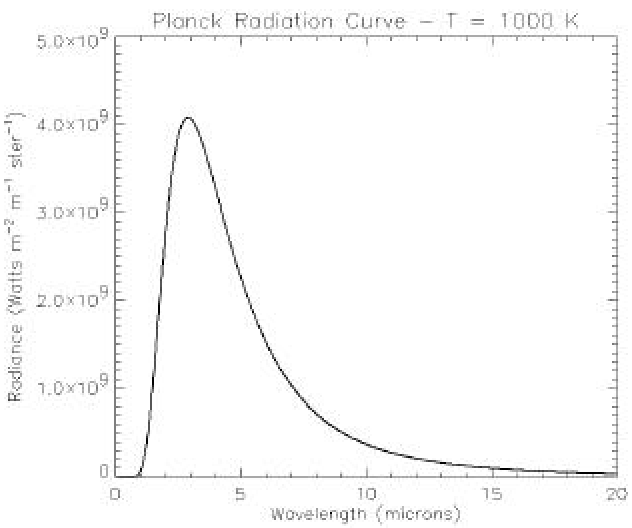
6. Calculate the radiance, L(λ), for T = 1000 K, from λ=0-20μ, and plot. Note that this is an exercise in calculation, and you should be sure you can obtain the correct answer with a hand calculator at a minimum of 2 wavelengths - say 3 and 10 μ.



note that I have used purely metric units here, vice the inverse microns in the text illustration.

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7. Calculate the peak wavelength for radiation at T=297 K, 1000 K, and 5800 K, in microns and nano-meters.

$$l_{\max} = \frac{a}{T} = \frac{2.9\times10^{-3}}{T}$$

= 9.73, 2.89, and 0.498 microns, respectively. The 2.89 micron peak is the location of the peak in the above figure.

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8. Calculate the radiated power for a black body at 297 K, in Watts/m^2.

Radiated Power = $\varepsilon\ \sigma\ T^4$ where we are taking ε = 1 in the absence of any other information.

Radiated Power = $1 \bullet 5.67\times10^{-8} \bullet 297^4 = 441.2\ \frac{\text{Watts}}{\text{m}^2}$

If your body's surface area is about 0.25 m^2, then you are producing the same amount of heat as a 100 Watt light bulb.

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9. Calculate the radiated power for a gray body at 297 K, ε=0.8, in Watts/m^2. Assume a surface area of 2 m^2, and calculate the radiated power, in Watts

Radiated Power = $0.8 \bullet 5.67\times10^{-8} \bullet 297^4 = 353\ \frac{\text{Watts}}{\text{m}^2}$

For a surface area of 2 m^2, this is 706 Watts

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10. Snell's law in optics is normally given as:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Where the angles are defined as in Figure 2.10. For an air/water interface, one can use the typical values: $n_1 = 1;\ n_2 = 1.33$. For such values, calculate θ_2 if $\theta_1 = 30^\circ$. What is the speed of light in water?

$$\sin\theta_2 = \frac{\sin 30^o}{1.33} = \frac{0.5}{1.33} = 0.376$$

$$\theta_2 = 22.1^o$$

$$v = \frac{c}{n} = 2.256\times10^8\ \frac{\text{m}}{\text{s}}$$